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Gauge-Yukawa Unification in Asymptotically Non-free Theories

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Abstract

We study asymptotically non-free gauge theories and search for renormalization group invariant (i.e. technically natural) relations among the couplings which lead to successful gauge-Yukawa unification. To be definite, we consider a supersymmetric model based on $SU(4) \times SU(2)_R \times SU(2)_L$. It is found that among the couplings of the model, which can be expressed in this way by a single one in the lowest order approximation, are the tree gauge couplings and the Yukawa coupling of the third generation. The corrections to the lowest order results are computed, and we find that the predictions on the low energy parameters resulting from those relations are in agreement with the measurements at LEP and Tevatron for a certain range of supersymmetry breaking scale.

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1 Introduction

The success of the standard model shows that we have at hand a highly nontrivial part of a more fundamental theory of elementary particle physics, and it challenges theorists to understand at least some of the plethora of its free parameters.

The well-known unification attempts [1, 2] assume that all gauge interactions are unified at a certain energy scale beyond which they are described by a unified gauge theory based on a simple gauge group–Grand Unified Theory (GUT). This unification idea has been not only inspiring for particle physicists, but also has given specific testable predictions [3]. The accurate measurements of the gauge couplings at LEP in fact suggest that the minimal $N = 1$ supersymmetric $SU(5)$ GUT [4] is very when comparing its theoretical values with the experiments [5].

GUTs can also relate Yukawa couplings among themselves which can lead to the prediction of fermion mass ratios. In the case of the minimal $SU(5)$ GUT [1], for instance, the prediction for the third generation, i.e., M_τ/M_b, was successful [6]. However, the GUT idea alone cannot provide us with the possibility to relate the gauge and Yukawa couplings. In order to achieve gauge-Yukawa coupling unification, within the assumption that all the particles appearing in a theory are elementary, one has to consider extended supersymmetric theories [7] or string theories [8]. Unfortunately, these theories seem to introduce more serious and difficult phenomenological problems to be solved than those of the standard model.

Here we would like to emphasize an alternative way to achieve unification of couplings [9]-[15] which is based on the fact that within the framework of renormalizable field theory, one can find renormalization group invariant (RGI) relations among parameters which can improve the calculability and the predictive power of the theory. These relations could in principle involve all the couplings of the theory, and this field theory technique is sometimes called "reduction of couplings" [10]. Along the RGI approach, there exists already studies and also certain success [12]-[15]. In refs. [14, 15], we have found that the gauge and Yukawa couplings in supersymmetric $SU(5)$ models can be unified using this method, which are consistent with the known experimental facts including the CDF

result on the top quark mass [16]. Moreover, the model proposed in ref. [14] is finite in the sense that all the β-functions vanish to all orders in perturbation theory [17].

Clearly, in both cases we have assumed the existence of a covering GUT so that the unification of the gauge couplings of the standard model is of a group theoretic nature. In this letter, we would like to examine the power of the RGI method by considering theories without covering GUTs.

It turns out that, in order the RGI method for the gauge coupling unification to work, the gauge couplings should have the same asymptotic behavior either in the ultraviolet or infrared regime. Unfortunately, this common behavior does not appear in the standard model with three families, since $SU(3)_C$ and $U(1)_Y$ couplings have opposite asymptotic behavior. One can increase the number of generations to make the $SU(3)_C$ and $SU(2)_L$ couplings also asymptotically non-free [18, 19]. But we prefer not to introduce new relatively light degrees of freedom, although we are in sympathy with this approach to non-perturbative unification. Another way to achieve a common asymptotic behavior of all the different gauge couplings is to embed the $SU(3)_C \times SU(2)_L \times U(1)_Y$ to some non-abelian gauge group which is not a simple group. That is, we introduce new physics at a very high energy scale and increase the predictability of the model on the known physics by using the RGI method. It turns out that the minimal phenomenologically viable model is based on the gauge group of Pati and Salam [20]– $\mathcal{G}_{\rm PS} \equiv SU(4) \times SU(2)_R \times SU(2)_L$ which is asymptotically non-free if it is supersymmetrized in a realistic fashion.. We would like to recall that $N = 1$ supersymmetric models based on this gauge group have been studied with renewed interest because they could in principle be derived from superstring [21, 22].

2 The model

Our supersymmetric gauge model is based on the gauge group $\mathcal{G}_{\rm PS}$, and we follow the definition of ref. [22] for the electric charge Q and the weak hypercharge Y:

$$Q \;=\; Y + \frac{1}{2} T_L \ , \ Y \;=\; \frac{1}{6} T_{15} + \frac{1}{2} T_R \ , \tag{1}$$

where $T_{15} = \text{diag.}\,(1,1,1,-3)$ and $T_{R,L} = \text{diag.}\,(1,-1)$. Three generations of quarks and leptons can be accommodated by six chiral supermultiplets, three in $(\mathbf{4},\mathbf{2},\mathbf{1})$ and three $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$ of $\mathcal{G}_{\rm PS}$, which we denote by $\Psi^{(I)\mu\ i_R}$ and $\overline{\Psi}^{(I)i_L}_{\mu}$, respectively. Here I runs over the three generations, and $\mu,\nu\ (=1,2,3,4)$ are the $SU(4)$ indices while i_R , i_L $(=1,2)$ stand for the $SU(2)_{L,R}$ indices. The model also consists of Higgs supermultiplets in $(\mathbf{4},\mathbf{2},\mathbf{1})$, $(\overline{\mathbf{4}},\mathbf{2},\mathbf{1})$ and $(\mathbf{15},\mathbf{1},\mathbf{1})$ of $\mathcal{G}_{\rm PS}$, $H^{\mu\ i_R}$, $\overline{H}_{\mu\ i_R}$ and Σ^{μ}_{ν}, respectively. They are responsible for the spontaneous symmetry breaking (SSB) of $SU(4)\times SU(2)_R$ down to $SU(3)_C\times U(1)_Y$. The SSB of $U(1)_Y\times SU(2)_L$ is then achieved by the nonzero VEV of $h_{i_R i_L}$ which is in $(\mathbf{1},\mathbf{2},\mathbf{2})$ of $\mathcal{G}_{\rm PS}$. In addition to these Higgs supermultiplets, we introduce $G^{\mu}_{\nu\ i_R i_L}$ $(\mathbf{15},\mathbf{2},\mathbf{2})$, ϕ $(\mathbf{1},\mathbf{1},\mathbf{1})$ and Σ'^{μ}_{ν} $(\mathbf{15},\mathbf{1},\mathbf{1})$. $G^{\mu}_{\nu\ i_R i_L}$ is introduced to realize the $SU(4)\times SU(2)_R\times SU(2)_L$ version of the Georgi-Jarlskog type ansatz [23] for the mass matrix of leptons and quarks while ϕ is supposed to mix with the right-handed neutrino supermultiplets at a high energy scale. The rôle of Σ'^{μ}_{ν} will be clear later on.

The superpotential of the model is given by

$$W = W_Y + W_{GJ} + W_{NM} + W_{AB} + W_{TDS} + W_M \ , \tag{2}$$

where

$$\begin{aligned}
W_Y &= \sum_{I,J=1}^{3} g_{IJ}\,\overline{\Psi}^{(I)i_R}_{\mu}\,\Psi^{(J)\mu\ i_L}\ h_{i_R i_L}\ ,\ W_{GJ} \;=\; g_{GJ}\,\overline{\Psi}^{(2)i_R}_{\mu}\,G^{\mu}_{\nu\ i_R j_L}\,\Psi^{(2)\nu\ j_L}\ ,\\
W_{NM} &= \sum_{I=1,2,3} g_{I\phi}\ \epsilon_{i_R j_R}\,\overline{\Psi}^{(I)i_R}_{\mu}\ H^{\mu\ j_R}\,\phi\ ,\\
W_{SB} &= g_H\,\overline{H}_{\mu\ i_R}\,\Sigma^{\mu}_{\nu}\,H^{\nu\ i_R} + \frac{g_\Sigma}{3}\,\mathrm{Tr}\ [\ \Sigma^3\ ] + \frac{g_{\Sigma'}}{2}\,\mathrm{Tr}\ [\ (\Sigma')^2\,\Sigma\ ]\ ,\\
W_{TDS} &= \frac{g_G}{2}\,\epsilon^{i_R j_R}\epsilon^{i_L j_L}\,\mathrm{Tr}\ [\ G_{i_R i_L}\,\Sigma\,G_{j_R j_L}\ ]\ ,\\
W_M &= m_h\,h^2 + m_G\,G^2 + m_\phi\,\phi^2 + m_H\,\overline{H}\,H + m_\Sigma\,\Sigma^2 + m_{\Sigma'}\,(\Sigma')^2\ .
\end{aligned} \tag{3}$$

Although the superpotential has the parity, $\phi\to-\phi$ and $\Sigma'\to-\Sigma'$, it is not the most general potential, and, by virtue of the nonrenormalization theorem, this does not contradict the philosophy of the coupling unification by the RGI method. W_{SB} is responsible for the $SU(4)\times SU(2)_R\to SU(3)_C\times U(1)_Y$ breaking, and it is achieved by the nonzero VEVs

$$< H^{41} > \;=\; < H_{41} > \;=\; v_H\ ,\ \ < \Sigma^{\alpha}_{\beta} > \;=\; \text{diag.}\,(1,1,1,-3)\,v_\Sigma\ , \tag{4}$$

in such a way that supersymmetry remains unbroken. This scale is expected to be of $O(M_{GUT})$. It is then easy to see that the right-handed neutrinos become heavy through W_{NM} after the SBB above [21].

The Yukawa couplings for leptons and quarks are contained in W_Y and W_{GJ}, where W_{GJ} is introduced to provide the Georgi-Jarlskog type ansatz [23]. So $T_{\nu}^{15\ \mu} G^{\nu}_{\mu\ i_R i_L}$ must be relatively light. We assume that the other components, leptoquarks and colored particles, are $O(M_{GUT})$, and that the superpotential W_{TDS} can realize this "triplet-doublet" splitting of G. To realize the SSB down to $SU(3)_C \times U(1)_{EM}$, we assume that there exists a choice of soft supersymmetry breaking terms so that the VEVs

$$\begin{aligned} < h_{i_R i_L} > &= \delta_{i_R,1}\delta_{i_L,2}\, v_D + \delta_{i_R,2}\delta_{i_L,1}\, v_U \ , \\ < G^{\alpha}_{\beta\ i_R i_L} > &= \text{diag.}\ (1,1,1,-3)\,(\,\delta_{i_R,1}\,\delta_{i_L,2}\, v_{GD} + \delta_{i_R,2}\,\delta_{i_L,1}\, v_{GU}\,) \end{aligned} \tag{5}$$

really corresponds to the minimum of the potential.

Given the supermultiplet content and the superpotential (2), it is now possible to compute the β-functions of the model. We denote the gauge couplings of $SU(4)\times SU(2)_R \times SU(2)_L$ by g_4 , g_{2R} and g_{2L}, respectively. The gauge coupling for $U(1)_Y$, g_1, normalized in the usual GUT inspired manner, is a function of them:

$$\frac{1}{g_1^2} = \frac{2}{5g_4^2} + \frac{3}{5g_{2R}^2}\ . \tag{6}$$

Normalizing the one-loop β-functions as $dg_i/d\ln\mu \ = \ \beta_i^{(1)} + O(g^5)$, $i = 2L, 2R, \cdots, \Sigma', G$, where μ is the renormalization scale, we find:

$$\begin{aligned} \beta^{(1)}_{2L} &= \frac{g_{2L}^3}{16\pi^2}\,16\ ,\ \beta^{(1)}_{2R} \ = \ \frac{g_{2R}^3}{16\pi^2}\,20\ ,\ \beta^{(1)}_{4} \ = \ \frac{g_4^3}{16\pi^2}\,18\ , \\ \beta^{(1)}_{GJ} &= \frac{g_{GJ}}{16\pi^2}\,[\,16|g_{GJ}|^2 + |g_{2\phi}|^2 + \frac{3}{2}|g_G|^2 - \frac{31}{2}|g_4|^2 - 3|g_{2R}|^2 - 3|g_{2L}|^2\,]\ , \\ \beta^{(1)}_{33} &= \frac{g_{33}}{16\pi^2}\,[\,8|g_{33}|^2 + |g_{3\phi}|^2 - \frac{15}{2}|g_4|^2 - 3|g_{2R}|^2 - 3|g_{2L}|^2\,]\ , \\ \beta^{(1)}_{1\phi} &= \frac{g_{1\phi}}{16\pi^2}\,[\,9\sum_{I=1}^{3}|g_{I\phi}|^2 + |g_{1\phi}|^2 + \frac{15}{4}|g_H|^2 - \frac{15}{2}|g_4|^2 - 3|g_{2R}|^2\,]\ , \\ \beta^{(1)}_{2\phi} &= \frac{g_{2\phi}}{16\pi^2}\,[\,9\sum_{I=1}^{3}|g_{I\phi}|^2 + |g_{2\phi}|^2 + \frac{15}{2}|g_{GJ}|^2 + \frac{15}{4}|g_H|^2 - \frac{15}{2}|g_4|^2 - 3|g_{2R}|^2\,]\ , \\ \beta^{(1)}_{3\phi} &= \frac{g_{3\phi}}{16\pi^2}\,[\,9\sum_{I=1}^{3}|g_{I\phi}|^2 + |g_{3\phi}|^2 + \frac{15}{4}|g_H|^2 + 2|g_{33}|^2 - \frac{15}{2}|g_4|^2 - 3|g_{2R}|^2\,]\ , \end{aligned} \tag{7}$$

$$\beta_H^{(1)} = \frac{g_H}{16\pi^2}\,[\sum_{I=1}^{3}|g_{I\phi}|^2+\frac{19}{2}|g_H|^2+3|g_\Sigma|^2+\frac{3}{4}|g_{\Sigma'}|^2+3|g_G|^2-\frac{31}{2}|g_4|^2-3|g_{2R}|^2\,]\ ,$$

$$\beta_\Sigma^{(1)} = \frac{g_\Sigma}{16\pi^2}\,[\,9|g_\Sigma|^2+6|g_H|^2+\frac{9}{4}|g_{\Sigma'}|^2+9|g_G|^2-24|g_4|^2\,]\ ,$$

$$\beta_{\Sigma'}^{(1)} = \frac{g_{\Sigma'}}{16\pi^2}\,[\,3|g_\Sigma|^2+2|g_H|^2+\frac{15}{4}|g_{\Sigma'}|^2+3|g_G|^2-24|g_4|^2\,]\ ,$$

$$\beta_G^{(1)} = \frac{g_G}{16\pi^2}\,[\,2|g_{GJ}|^2+3|g_\Sigma|^2+2|g_H|^2+\frac{3}{4}|g_{\Sigma'}|^2+6|g_G|^2-24|g_4|^2-3|g_{2R}|^2-3|g_{2L}|^2\,]\ .$$

We have assumed that the Yukawa couplings g_{IJ} except for g_{33} vanish. They can be included into RGI relations as small perturbations [1], but we assume here that their numerical effects will be negligibly small, so that we will suppress them in the following discussions.

3 Gauge-Yukawa-Higgs unification by the RGI method

Any RGI relation among couplings can be expressed in the implicit form $\Phi(g_1,\cdots,g_N) = \text{const.}$, which has to satisfy the partial differential equation

$$\vec{\beta}\cdot\vec{\nabla}\,\Phi = \sum_{i=1}^{N}\beta_i\,\frac{\partial}{\partial g_i}\,\Phi = 0\ , \tag{8}$$

where β_i is the β-function of g_i ($i = 1,\cdots,N$). If the β-functions satisfy a certain regularity, there exist, at least locally, $(N-1)$ independent solutions of (8), and they are equivalent to the solutions to the ordinary differential equations, the so-called reduction equations [10],

$$\beta\,\frac{dg_i}{dg} = \beta_i\ ,\ i=1,\cdots,N\ , \tag{9}$$

where g and β are the primary coupling and its β-function, and i does not include it. Since maximally $N-1$ independent RGI "constraints" in the N-dimensional space of couplings can be imposed by Φ_i, one could in principle express all the couplings in terms of a single coupling, the primary coupling g [10]. This possibility is without any doubt attractive, but it can be unrealistic. Therefore, one often would like to impose fewer RGI constraints, and this is the idea of partial reduction [11, 13]. From this point of view,

[1]The meaning of the small perturbations will be clarified later on.

the partial differential equation (8) can provide us with an intuitive picture of partial reduction, though both differential equations (8) and (9) are mathematically equivalent.

Detailed discussions on partial reduction are given in ref. [15] for instance, and here we would like to briefly outline the method. For the case at hand, it is convenient to work with the absolute square of g_i, and we define the tilde couplings by

$$\tilde{\alpha}_i \equiv \frac{\alpha_i}{\alpha} \ , \ i = 1, \cdots, N \ , \tag{10}$$

where $\alpha = |g|^2/4\pi$ and $\alpha_i = |g_i|^2/4\pi$ (i does not include the primary coupling). We assume that their evolution equations take the form

$$\begin{aligned} \frac{d\alpha}{dt} &= -b^{(1)} \alpha^2 + \cdots \ , \\ \frac{d\alpha_i}{dt} &= -b_i^{(1)} \alpha_i \alpha + \sum_{j,k} b_{i,jk}^{(1)} \alpha_j \alpha_k + \cdots \ , \end{aligned}$$

in perturbation theory, and then derive

$$\alpha \frac{d\tilde{\alpha}_i}{d\alpha} = (-1 + \frac{b_i^{(1)}}{b^{(1)}}) \tilde{\alpha}_i - \sum_{j,k} \frac{b_{i,jk}^{(1)}}{b^{(1)}} \tilde{\alpha}_j \tilde{\alpha}_k + \sum_{r=2} (\frac{\alpha}{\pi})^{r-1} \tilde{b}_i^{(r)}(\tilde{\alpha}) \ , \tag{11}$$

where $\tilde{b}_i^{(r)}(\tilde{\alpha})$ $(r = 2, \cdots)$ are power series of $\tilde{\alpha}_i$ and can be computed from the r-th loop β-functions.

To procced, we have to solve the set of the algebraic equations

$$(-1 + \frac{b_i^{(1)}}{b^{(1)}}) \rho_i - \sum_{j,k} \frac{b_{i,jk}^{(1)}}{b^{(1)}} \rho_j \rho_k = 0 \ , \tag{12}$$

and assume that the solutions ρ_i's have the form

$$\rho_i = 0 \text{ for } i = 1, \cdots, N' \ ; \ \rho_i > 0 \text{ for } i = N'+1, \cdots, N \ . \tag{13}$$

We then regard $\tilde{\alpha}_i$ with $i \leq N'$ as small perturbations to the undisturbed system which is defined by setting $\tilde{\alpha}_i$ with $i \leq N'$ equal to zero.. We recall that it is possible [10] to verify at the one-loop level the existence of the unique power series solutions

$$\tilde{\alpha}_i = \rho_i + \sum_{r=2} \rho_i^{(r)} (\frac{\alpha}{\pi})^{r-1} \ , \ i = N'+1, \cdots, N \tag{14}$$

of the reduction equations (11) to all orders in the undisturbed system . These are RGI relations among couplings and keep formally perturbative renormalizability of the

undisturbed system. So in the undisturbed system there is only *one independent* coupling, the primary coupling α.

The small perturbations caused by nonvanishing $\tilde{\alpha}_i$ with $i \leq N'$ enter in such a way that the reduced couplings, i.e., $\tilde{\alpha}_i$ with $i > N'$, become functions not only of α but also of $\tilde{\alpha}_i$ with $i \leq N'$. It turned out that, to investigate such partially reduced systems, it is most convenient to work with the partial differential equations

$$\begin{aligned} \{ \ \tilde{\beta}\,\frac{\partial}{\partial \alpha} + \sum_{a=1}^{N'} \tilde{\beta}_a\,\frac{\partial}{\partial \tilde{\alpha}_a} \ \} \ \tilde{\alpha}_i(\alpha,\tilde{\alpha}) &= \tilde{\beta}_i(\alpha,\tilde{\alpha}) \ , \\ \tilde{\beta}_{i(a)} = \frac{\beta_{i(a)}}{\alpha^2} - \frac{\beta}{\alpha^2}\,\tilde{\alpha}_{i(a)} \quad &, \quad \tilde{\beta} \equiv \frac{\beta}{\alpha} \ , \end{aligned} \tag{15}$$

which are equivalent to the reduction equations (11), where we let a, b run from 1 to N' and i, j from $N'+1$ to N, in order to avoid confusion. We then look for solutions of the form

$$\tilde{\alpha}_i = \rho_i + \sum_{r=1} (\frac{\alpha}{\pi})^{r-1}\, f_i^{(r)}(\tilde{\alpha}_a) \ , \ i = N'+1, \cdots, N \ , \tag{16}$$

where $f_i^{(r)}(\tilde{\alpha}_a)$ are supposed to be power series of $\tilde{\alpha}_a$. This particular type of solution can be motivated by requiring that in the limit of vanishing perturbations we obtain the undisturbed solutions (14) [13, 24], i.e., $f_i^{(1)}(0) = 0$, $f_i^{(r)}(0) = \rho_i$ for $r \geq 2$. Again it is possible to obtain the sufficient conditions for the uniqueness of $f_i^{(r)}$ in terms of the lowest order coefficients.

With these discussions above in mind, we would like to present our results for the present model below. In principle, the primary coupling can be any one of the couplings. But it is more convenient to choose a gauge coupling as the primary one because the one-loop β functions for a gauge coupling depends only on its own gauge coupling. For the present model, we use α_{2L} as the primary one.

(i) Gauge sector

Since the gauge sector at the one-loop β functions is closed as said, the solutions of the fixed point equations (12) are independent on the Yukawa and Higgs couplings. One easily obtains

$$\rho_4 = \frac{8}{9} \ , \ \rho_{2R} = \frac{4}{5} \ , \tag{17}$$

where we have used the one-loop β- functions (7) in the gauge sector and eq. (12). Using now eq. (6), we find that the RGI relations (14) become

$$\begin{aligned} \tilde{\alpha}_4 &= \frac{\alpha_4}{\alpha_{2L}} = \frac{8}{9} \,,\; \tilde{\alpha}_1 = \frac{\alpha_1}{\alpha_{2L}} = \frac{5}{6} \,, \\ \sin^2\theta_W &= \frac{3\alpha_1/5\alpha_{2L}}{1+3\alpha_1/5\alpha_{2L}} = \frac{1}{3} \,. \end{aligned} \tag{18}$$

Furthermore, one can convince oneself that at the one-loop level there is no correction to eq. (18) which can result from perturbations to the undisturbed system. The RGI relations (18) are also boundary conditions at M_{GUT}, where, at M_{GUT}, the QCD coupling α_S can be identified with α_4.

(ii) Yukawa-Higgs sector

The solutions of eq. (12) in the Yukawa-Higgs sector strongly depend on the result of the gauge sector. Since there are 9 couplings in this sector, eq. (12) could in principle admit $2^9 = 512$ independent solutions. But solutions with negative ρ cannot be accepted because α_i and the primary coupling $\alpha = \alpha_{2L}$ are positive semidefinite (see eq. (10)). Note also that the more vanishing ρ_i's a solution contains, the less is its predictive power. After slightly involved algebraic computations, one finds that most predictive solutions contain at least three vanishing ρ_i's. There exist 11 solutions of that type, but their predictive power on low energy parameters is not equally significant. Out of these 11 solutions, there are two, A and B, that satisfy

$$\rho_{33} \;,\; \rho_{GJ} \;>\; 0 \;\text{ and } \rho_{3\phi} \;>\; \rho_{1\phi} \;, \rho_{2\phi} \;. \tag{19}$$

These contain RGI relations that exhibit the most predictive power and moreover they satisfy the neutrino mass relation $M_{\nu_\tau} > M_{\nu_\mu} \,,\, M_{\nu_e}$.

For the solution A, we have $\rho_{1\phi} = \rho_{2\phi} = \rho_\Sigma = 0$, while for the solution B, $\rho_{1\phi} = \rho_{2\phi} = \rho_G = 0$, and the rest of the ρ_i's are given by

$$\begin{aligned} \rho_{GJ} &= \begin{cases} 289721/173010 & \simeq 1.67 \\ 1583/720 & \simeq 2.20 \end{cases} \,,\; \rho_{33} = \begin{cases} 1151909/346020 & \simeq 3.33 \\ 7543/2220 & \simeq 3.40 \end{cases} \,, \\ \rho_{3\phi} &= \begin{cases} 41363/28835 & \simeq 1.43 \\ 491/555 & \simeq 0.88 \end{cases} \,,\; \rho_H = \begin{cases} 93746/86505 & \simeq 1.08 \\ 6974/2775 & \simeq 2.51 \end{cases} \,, \end{aligned} \tag{20}$$

$$\rho_\Sigma = \begin{cases} 0 \\ 9956/24975 \quad \simeq 0.40 \end{cases}, \quad \rho_{\Sigma'} = \begin{cases} 3819496/778545 & \simeq 4.91 \\ 224/27 & \simeq 8.30 \end{cases},$$
$$\rho_G = \begin{cases} 4351714/778545 \quad \simeq 5.59 \\ 0 \end{cases} \quad \text{for} \quad \begin{cases} A \\ B \end{cases}.$$

The corrections to the above RGI relations in the lowest order in the undisturbed system, which come from the perturbations, can be computed, and one finds in the first order

$$\begin{aligned}
\tilde{\alpha}_{GJ} &\simeq \begin{cases} 1.67 - 0.05\tilde{\alpha}_{1\phi} + 0.004\tilde{\alpha}_{2\phi} - 0.90\tilde{\alpha}_\Sigma + \cdots \\ 2.20 - 0.08\tilde{\alpha}_{2\phi} - 0.05\tilde{\alpha}_G + \cdots \end{cases}, \\
\tilde{\alpha}_{33} &\simeq \begin{cases} 3.33 + 0.05\tilde{\alpha}_{1\phi} + 0.21\tilde{\alpha}_{2\phi} - 0.02\tilde{\alpha}_\Sigma + \cdots \\ 3.40 + 0.05\tilde{\alpha}_{1\phi} - 1.63\tilde{\alpha}_{2\phi} - 0.001\tilde{\alpha}_G + \cdots \end{cases}, \\
\tilde{\alpha}_{3\phi} &\simeq \begin{cases} 1.43 - 0.58\tilde{\alpha}_{1\phi} - 1.43\tilde{\alpha}_{2\phi} - 0.03\tilde{\alpha}_\Sigma + \cdots \\ 0.88 - 0.48\tilde{\alpha}_{1\phi} + 8.83\tilde{\alpha}_{2\phi} + 0.01\tilde{\alpha}_G + \cdots \end{cases}, \\
\tilde{\alpha}_H &\simeq \begin{cases} 1.08 - 0.03\tilde{\alpha}_{1\phi} + 0.10\tilde{\alpha}_{2\phi} - 0.07\tilde{\alpha}_\Sigma + \cdots \\ 2.51 - 0.04\tilde{\alpha}_{1\phi} - 1.68\tilde{\alpha}_{2\phi} - 0.12\tilde{\alpha}_G + \cdots \end{cases}, \\
\tilde{\alpha}_\Sigma &\simeq \begin{cases} --- \\ 0.40 + 0.01\tilde{\alpha}_{1\phi} - 0.45\tilde{\alpha}_{2\phi} - 0.10\tilde{\alpha}_G + \cdots \end{cases}, \\
\tilde{\alpha}_{\Sigma'} &\simeq \begin{cases} 4.91 - 0.001\tilde{\alpha}_{1\phi} - 0.03\tilde{\alpha}_{2\phi} - 0.46\tilde{\alpha}_\Sigma + \cdots \\ 8.30 + 0.01\tilde{\alpha}_{1\phi} + 1.72\tilde{\alpha}_{2\phi} - 0.36\tilde{\alpha}_G + \cdots \end{cases}, \\
\tilde{\alpha}_G &\simeq \begin{cases} 5.59 + 0.02\tilde{\alpha}_{1\phi} - 0.04\tilde{\alpha}_{2\phi} - 1.33\tilde{\alpha}_\Sigma + \cdots \\ --- \end{cases} \quad \text{for} \quad \begin{cases} A \\ B \end{cases}.
\end{aligned} \tag{21}$$

Note that $\tilde{\alpha}_{GJ}$ is in the same order of magnitude as $\tilde{\alpha}_{33}$ for both solutions and the masses of the second and third fermion generations are approximately proportional to $\sqrt{\tilde{\alpha}_{GJ}}$ and $\sqrt{\tilde{\alpha}_{33}}$, respectively. Therefore, we must require that

$$v_D \gg v_{GD} \quad \text{and} \quad v_U \gg v_{GU} \tag{22}$$

to satisfy the observed fermion mass hierarchy, where VEVs are defined in eq. (5). Consequently, we will neglect in the following numerical analysis the contributions of v_{GD} and v_{GU} to the top and bottom quark and tau masses (and also to M_Z).

4 Results and discussions

Until now we have assumed that supersymmetry is unbroken. But we would like to recall that the RGI relations (18) and (21) we have obtained above remain unaffected by dimensional parameters in mass-independent renormalization schemes such as the minimal subtraction (MS) scheme. Therefore, those RGI relations have still their validity if supersymmetry breaking is soft.

The next step is to express the RGI relations (18) and (21) in terms of observable parameters. To this end, we apply the well-known renormalization group technique and regard the RGI relations as the boundary conditions holding at the unification scale M_{GUT} in addition to the group theoretic one $\alpha_{33} = \alpha_t \ = \ \alpha_b \ = \ \alpha_\tau$.

Just below the unification scale we would like to obtain the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model while requiring that all the superpartners are decoupled below the supersymmetry breaking scale M_{SUSY}. Then the standard model should be spontaneously broken down to $SU(3)_C \times U(1)_{EM}$ due to VEVs (5). We assume that the low energy theory which satisfies the requirement above can be obtained by arranging soft supersymmetry breaking terms and the mass parameters in the superpotential (2) in an appropriate fashion.

One of the large theoretical uncertainties after all the above is done is the arbitrariness of the superpartner masses. To simplify our numerical analysis we would like to assume a unique threshold M_{SUSY} for all the superpartners. Another one is the number of the light Higgs particles that are contained in $h_{i_R i_L}$ and also in $G^{\mu}_{\nu\ i_R i_L}$. The number of the Higgses lighter than M_{SUSY} (which we denote by N_H) namely could vary from one to four while the number of those to be taken into account above M_{SUSY} is fixed at four. After these remarks, we examine numerically the evolution of the gauge and Yukawa couplings including the two-loop effects, according to their renormalization group equations.

In table 1 we present the low energy parameters of the present model for three distinct boundary conditions; $\tilde{\alpha}_{33}(M_{GUT}) = 4.0$, 3.2 and 2.8 with $N_H = 1$. All the dimensionless parameters (except $\tan\beta$) are defined in the $\overline{\mathrm{MS}}$ scheme, and all the masses (except for M_{GUT}) are pole masses.

M_{SUSY} [TeV]	$\tilde{\alpha}_{33}(M_{GUT})$	$\alpha_S(M_Z)$	$\alpha(M_{GUT})$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	$M_t[GeV]$
1.6	4.0	0.119	0.046	63.0	0.9×10^{15}	5.01	197.8
1.6	3.2	0.119	0.046	63.0	0.9×10^{15}	4.97	196.1
1.6	2.8	0.119	0.046	63.0	0.9×10^{15}	4.95	195.1

Table 1. The predictions for different boundary conditions, where we have used: $M_\tau = 1.78$ GeV, $\alpha_{em}^{-1}(M_Z) = 127.9$ and $\sin_W(M_Z) = 0.2303$.

Note that the corrections to $\sin^2\theta_W(M_Z)$ that come from a large M_t, i.e., $\sin^2\theta_W(M_Z) = 0.2324 - 10^{-7}\,[138^2 - (M_t/\text{GeV})^2]$, are taken into account above and below. We see from table 1 that the low energy predictions are insensitive against the value of $\tilde{\alpha}_{33}$. The low energy predictions for various M_{SUSY} with fixed $\tilde{\alpha}_{33}$ are shown in table 2.

M_{SUSY} [TeV]	$\tilde{\alpha}_{33}(M_{GUT})$	$\alpha_S(M_Z)$	$\alpha(M_{GUT})$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	$M_t[GeV]$
1.3	3.2	0.117	0.046	63.4	0.8×10^{15}	4.82	194.5
3.4	3.2	0.110	0.044	63.0	0.5×10^{15}	4.69	193.6
4.4	3.2	0.112	0.044	64.2	0.6×10^{15}	4.74	195.3

Table 2. The predictions for different M_{SUSY} with fixed $\tilde{\alpha}_{33}$.

Except for M_{SUSY} all the quantities in the tables are predicted; the range of $\tilde{\alpha}_{33}$ is also given by the model (see eq. (21)). In fig. 1 we plot M_t versus M_{SUSY}. We see from the graph that there are no realistic solutions for low values of M_{SUSY} ($\sim M_Z - 300$ GeV) and the present model rather prefers large values of M_{SUSY} (> 400 GeV).

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References

[1] H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32** (1974) 438

[2] H. Fritzsch and P. Minkowski, Ann. Phys. **93** (1975) 193; H. Georgi, in *Particles and Fields–1974*, ed. C.E. Carlson, American Institute of Physics, New York

[3] H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. **33** (1974) 451

[4] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150; N. Sakai, Z. Phys. **C11** (1981) 153

[5] K. Inoue et al., Prog. Theor. Phys. **67** (1982) 1889; **68** (1982) 927; U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. **B260** (1991) 447

[6] A. Buras, J. Ellis, M. Gaillard and D. Nanopoulos, Nucl. Phys. **B135** (1978) 66

[7] P. Fayet, Nucl. Phys. **B149** (1979) 134; F. del Aguila, M. Dugan, B. Grinstein, L. Hall, G.G. Ross and P. West, Nucl. Phys. **B250** (1985) 225

[8] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory Vol. I,II*, Cambridge UP, Cambridge, 1987; D. Lüst and S. Theisen, *Lectures on String Theory*, Springer Notes in Physics, Vol. 346, Heidelberg, 1989

[9] N.P. Cheng, Phys. Rev. **D10** (1974) 2706

[10] W. Zimmermann, Commun. Math. Phys. **97** (1985) 211; R. Oehme and W. Zimmermann, Commun. Math. Phys. **97** (1985) 569; R. Oehme, Prog. Theor. Phys. Suppl. **86** (1986) 215

[11] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. **B259** (1985) 331

[12] K. Sibold and W. Zimmermann, Phys. Lett. **B191** (1987) 427; A. Denner, Nucl. Phys. **B347** (1990) 184

[13] J. Kubo, K. Sibold and W. Zimmermann, Phys. Lett. **B200** (1989) 185; J. Kubo, Phys. Lett. **B262** (1991) 472

[14] D. Kapetanakis, M. Mondragón and G. Zoupanos, Z. Phys. **C60** (1993) 181; M. Mondragón and G. Zoupanos, $SU(5)$ *Finite Unified Theories and the mass of the top quark*, CERN preprint, CERN-TH.7098/93

[15] J. Kubo, M. Mondragon and G. Zoupanos, *Reduction of couplings and heavy top quark*, Max-Plank-Institute preprint, to appear in Nucl. Phys. **B**

[16] F. Abe et al., *Evidence for top quark production in* $p\overline{p}$ *collisions at* $\sqrt{s} = 1.8\ TeV$, Fermilab preprint, FERMILAB-PUB-94-116-E

[17] C. Lucchesi, O. Piguet and K. Sibold, Helv. Phys. Acta **61** (1988) 321

[18] L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. **B136** (1978) 115; N. Cabibo and G.R. Farrar, Phys. Lett. **B110** (1982) 107; L. Maiani and R. Petronzio, Phys. Lett. **B176** (1986) 12; G. Grunberg, Phys. Rev. Lett. **58** (1987) 1180; S. Theisen, N.D. Tracas and G. Zoupanos, Z. Phys. **C37** (1988) 597; C. Alacoque, C. Deom and J. Pestieau, Phys. Lett. **B228** (1989) 370; D. Kapetanakis, S. Theisen and G. Zoupanos, Phys. Lett. **B229** (1989) 248; P.Q. Hung, Phys. Rev. **D38** (1988) 377; J.P. Derendinger, R. Kaiser and M. Roncadelli, Phys. Lett. **B220** (1989) 164; G. Leontaris, C.E. Vayonakis and N.D. Tracas, Mod. Phys. Lett. **A4** (1989) 2429

[19] T. Mori, H. Murayama and T. Yanagida, Phys. Rev. **D48** (1993) 2995; B. Brahmachari, U. Sarkar and K. Sridhar, Mod. Phys. Lett. **A8** (1993) 3349

[20] J.C. Pati and A. Salam, Phys. Rev. Lett. **31** (1973) 661

[21] I. Antoniadis and G.K. Leontaris, Phys. Lett. **B216** (1989) 333

[22] G. Leontaris and N. Tracas, Z. Phys. **C56** (1992) 479; Phys. Lett. **B291** (1992) 44

[23] H. Georgi and C. Jarlskog, Phys. Lett. **B86** (1979) 297

[24] W. Zimmermann, Phys. Lett. **B311** (1993) 249

Figure Captions

Fig. 1. The M_{SUSY} dependence of the M_t prediction for $\tilde{\alpha}_{33}(M_{GUT}) = 3.2$.